Building Temperature Distributed Control via Explicit MPC and “Trim and Respond” Methods

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Abstract—We study the distributed control of heating, ventilation, and air conditioning systems in buildings. We present and compare two control design techniques. In the first part, a one-step distributed model predictive controller (DMPC) is introduced. We compute the explicit state-feedback solution and show how to implement it on existing networked buildings control platforms. In the second part, we discuss a control logic currently used in the buildings industry called “Trim and Respond”. We show that “Trim and Respond” controllers can be seen as a special class of one-step DMPC algorithms. We conclude the paper with a simulation study which shows the DMPC control design and its equivalence with “Trim and Respond” heuristics.

I. INTRODUCTION

The building sector consumes about 40% of the energy used in the United States and is responsible for nearly 40% of greenhouse gas emissions [1]. It is therefore economically, socially, and environmentally significant to reduce the energy consumption of buildings.

The main idea of predictive control is to use a model of the plant to predict the future evolution of the system [2], [3], [4]. At each sampling time, starting at the current state, an open-loop optimal control problem is solved over a finite horizon. The optimal command signal is applied to the process only during the following sampling interval. At the next time step, a new optimal control problem based on new measurements of the state is solved over a shifted horizon. The resultant control algorithm is referred to as Model Predictive Control (MPC).

In [5], [6], [7], [8], the authors have implemented MPC on building heating, ventilation, and air conditioning (HVAC) systems obtaining reduced peak power consumption and reduced overall energy usage compared to existing production logic. In the aforementioned literature, control computation is performed at a centralized unit. In practice, embedded building control platforms are physically distributed throughout the building. It is standard to have a central processor and router located at the main air supply location (called the Air Handling Unit) and an embedded control unit at the locations which regulate flow and temperature of each zone (called the Variable Air Volume box). The goal of this paper is to design distributed MPC control algorithms which fit these existing distributed control platforms.

Distributed Model Predictive Control (DMPC) in buildings with a large number of zones has been previously approached by using Sequential Quadratic Programming (SQP) and dual decomposition [9]. The control scheme is formulated for a nonlinear system using MPC and a generic prediction horizon N. The result is an iterative decentralized controller with substantial communication overhead.

In this paper, we present and compare two distributed control design approaches: a one-step distributed MPC and a heuristic control logic called “Trim and Respond”. “Trim and Respond” is a distributed control logic currently utilized in buildings [10]; it has been shown to be more efficient and is considered to be more intuitive to tune than control logic used by equipment manufacturers.

The paper is divided into two parts. In the first part, we compute the explicit solution of a one-step distributed MPC controller and show how to implement it on existing networked buildings control platforms. In the second part, we present the “Trim and Respond” logic and compare it with the explicit solution of a one-step DMPC. We show that “Trim and Respond” is a special class of one-step DMPC. The results of this paper can impact the way distributed building control logics are designed. On one hand, they allow the implementation of DMPC on existing distributed building control platforms with a small amount of communication; on the other hand, they can be used to improve “Trim and Respond” logic by decreasing the amount of manual parameter tuning.

The paper is organized in the following way. First, we introduce the system model in Section II. Then, the constrained finite time optimization problem for MPC is formulated in Section III. The distributed explicit solution is derived from KKT conditions in Section IV. “Trim and Respond” logic is introduced in Section V. The DMPC and “Trim and Respond” methods are compared in Section VI, a simulation of both logics is shown in Section VII, and finally, conclusions are drawn in Section VIII.

II. SYSTEM MODEL

This section introduces the components of a typical heating, ventilation, and air conditioning system. Air Handling Units (AHU) and Variable Air Volume (VAV) boxes are the main components used to heat or cool and distribute air in a building (Figure 1). The AHU recirculates return air from building spaces, and mixes it with fresh outside air. The proportion of return air to outside air is controlled by damper positions in the AHU (Figure 1). The mixed air can be cooled by cooling coils that extract the cooling energy from chilled water produced by chillers.

The air temperature after these coils depends on the coil valve positions, the temperature of the chilled or heated
Also, local actuation can only be in the form of heating and central actuation can only be in the form of cooling:

\[ u_c(k) \geq 0, \quad u_i(k) \leq 0. \quad (4) \]

Input saturation of \( u_c(k) \) and \( u_i(k) \) is not considered in this paper. The approach presented here can be extended to include these additional constraints.

III. MODEL PREDICTIVE CONTROL

In this section, a model predictive controller is designed to minimize the total energy consumption of the HVAC system defined in Section II, and to satisfy the thermal comfort constraints in equation (3) and the reheating and cooling constraints (4). Consider the following MPC problem at time step \( k \):

\[
\begin{align*}
\min_{u_c(k), u_i(k)} & \quad u_c(k)^2 + \sum_{i=1}^{n} u_i(k)^2 \\
\text{subject to} & \quad A_i T_i(k) + B_{C,i} u_c(k) + B_i u_i(k) + \hat{d}_i(k) \leq \bar{T} \quad (5b) \\
& \quad A_i T_i(k) + B_{C,i} u_c(k) + B_i u_i(k) + \hat{d}_i(k) \geq T \quad (5c) \\
& \quad u_c(k) \geq 0 \quad (5d) \\
& \quad u_i(k) \leq 0 \quad (5e) \\
& \quad \forall i \in \{1, \ldots, n\},
\end{align*}
\]

where the prediction horizon length is one, \( \hat{d}_i(k) \) is the estimated load at the \( i \)-th zone, (5b) and (5c) are thermal comfort constraints at the next time step \( k+1 \), (5d) is the local actuation reheating constraint, and (5e) is the central cooling constraint.

Let the optimal solution to problem (5) be

\[ U^* = \{u_c^*(k), u_1^*(k), \ldots, u_n^*(k)\}. \quad (6) \]

At time step \( k \), the solution \( u_c^*(k) \) is implemented by the AHU and \( u_i^*(k) \) is implemented by the \( i \)-th VAV box. The optimization (5) is repeated at time \( k+1 \), with the updated state estimation \( T_i(k+1) \) and estimated load \( d_i(k+1) \), yielding a receding horizon control strategy.

IV. EXPLICIT DISTRIBUTED MPC SOLUTION

The solution \( U^* \) in (6) can be computed by solving a quadratic program at each time step \( k \) for a given \( T_i(k), \hat{d}_i(k), T_{oa}(k), \hat{d}_{oa}(k) \). However, this would result in a centralized approach and would require software not readily available on current buildings embedded control systems.

Karush-Kuhn-Tucker (KKT) necessary and sufficient conditions are used to find the explicit expression of the optimal controller solution to problem (5). Compared to standard explicit MPC control design [12], our procedure provides a distributed explicit controller. The Lagrangian for problem (5) is

\[
\begin{align*}
L(u_c(k), u_1(k), \ldots, u_n(k), \lambda_c(k), \lambda_1(k), \ldots, \lambda_n(k), \\
\mu_1(k), \ldots, \mu_n(k), \mu_1(k), \ldots, \mu_n(k)) &= \\
& \quad u_c(k)^2 + \lambda_c(k)(u_c(k)) + \sum_{i=1}^{n} (u_i(k)^2 + \lambda_i(k)(-u_i(k)) \\
& \quad + \mu_1(k)(A_i T_i(k) + B_{C,i} u_c(k) + B_i u_i(k) + \hat{d}_i(k) - \bar{T}) \\
& \quad + \mu_1(k)(-A_i T_i(k) - B_{C,i} u_c(k) - B_i u_i(k) - \hat{d}_i(k) + T)),
\end{align*}
\]

Fig. 1: System Scheme
Three dual variables exist for each zone $i$: $\lambda_i(k)$ is associated with the non-negativity constraint on $u_i(k)$ (5d), $\bar{\mu}_i(k)$ is associated with the upper bound on the temperature $T_i(k+1)$ (5b), and $\check{\mu}_i(k)$ is associated with the lower bound on the temperature $\bar{T}_i(k+1)$ (5c). Also, $\lambda_i(k)$ corresponds to the non-positivity constraint on $u_c(k)$ (5e). The KKT conditions for problem (5) at time $k$ are:

\begin{align}
2u_{i}(k) + \sum_{i=1}^{n} B_{Ci,i}(\bar{\mu}_i - \check{\mu}_i) + \lambda_i(k) &= 0 \quad (8a) \\
2u_{i}(k) - \lambda_i(k) + B_i(\check{\mu}_i - \bar{\mu}_i) &= 0 \quad (8b) \\
\lambda_i(k)u_i(k) &= 0 \quad (8c) \\
\lambda_i(k)u_i(k) &= 0 \quad (8d) \\
\bar{\mu}_i(k)(A_i T_i(k) + B_{Ci}u_c(k) + B_i u_i(k) + \hat{d}_i(k) - \bar{T}) &= 0 \quad (8e) \\
\mu_i(k)(A_i T_i(k) + B_{Ci}u_c(k) + B_i u_i(k) + \hat{d}_i(k) - T) &= 0 \quad (8f) \\
u_{i}(k) &\geq 0 \quad (8g) \\
u_{i}(k) &\geq 0 \quad (8h) \\
A_i T_i(k) + B_{Ci}u_c(k) + B_i u_i(k) + \hat{d}_i(k) &\leq T \quad (8i) \\
A_i T_i(k) + B_{Ci}u_c(k) + B_i u_i(k) + \hat{d}_i(k) &\geq T \quad (8j) \\
\lambda_i(k) &\geq 0 \quad (8k) \\
\check{\mu}_i(k) &\geq 0 \quad (8l) \\
\bar{\mu}_i(k) &\geq 0 \quad (8m) \\
i \in \{1, \ldots, n\}, \\
\end{align}

where equations (8a) to (8b) are the components of the KKT condition $\nabla L(u_{i}(k), u_j(k), \ldots) = 0$, equations (8c) to (8f) are the complementary slackness conditions, equations (8g) to (8h) are the feasibility conditions, and equations (8k) to (8n) are the dual feasibility conditions.

In order to compute the feedback control law solving (8), we need to enumerate all possible combinations of active primal constraints. Intuition from physics makes this task easier. Only combinations of temperature bound constraints (8i) and (8j) need be considered, as the activeness of (8g) and (8h) are fully determined from active sets of (8i) and (8j). Let $I_H$ and $I_C$ be two subsets of the set $I = \{1, \ldots, n\}$. One can prove that only the following cases will generate a primal and dual feasible controller.

**Case I** Lower bound temperature constraints (8i) are active for all $i \in I_H$; upper bound temperature constraints (8i) are not active for all $i \in I$.

**Case II** Upper bound temperature constraints (8i) are active for all $i \in I_C$; lower bound temperature constraints (8i) are not active for all $i \in I$.

**Case III** Upper bound temperature constraints (8i) are active for all $i \in I_C$; lower bound temperature constraints (8i) are active for all $i \in I_H$.

**Case IV** Both temperature bound constraints (8i) and (8j) are not active for all $i \in I$.

Note that in Case III a single zone cannot simultaneously have an active upper temperature constraint and an active lower constraint, i.e. $I_H \cap I_C = \emptyset$ in Case III.

**Remark 1.** All primal variables, dual variables, and inputs are a function of time step $k$, but for readability this argument will be dropped from the notation for the remainder of this section.

The primal variables can be written as a function of the dual variables:

\begin{align}
u_c &= \frac{1}{2} \left( -\lambda_c + \sum_{i \in I} B_{Ci,i}(\mu_i - \check{\mu}_i) \right) \quad (9a) \\
u_i &= \frac{1}{2} \left( B_i(\mu_i - \bar{\mu}_i) + \lambda_i \right). \quad (9b)
\end{align}

Note that the optimal control law for Case IV is $u_c = u_i = 0 \forall i \in I$. The derivation of the optimal distributed controller for Case I is described next and a summary of the other cases are provided in the following subsections.

**A. Case I: Active Lower Bound Temperature Constraints and No Active Upper Bound Temperature Constraint**

In Case I we have:

\begin{align}
\check{\mu}_i &= 0 \forall i \in I \quad (10a) \\
\bar{\mu}_i &= 0 \forall i \in I \setminus I_H \quad (10b) \\
A_i T_i + B_{Ci}u_c + B_i u_i + \hat{d}_i &= T \forall i \in I_H. \quad (10c)
\end{align}

Simplifying (9a) by using complimentary slackness (8d) with dual feasibility (8k) and (8m), we obtain:

\begin{align}
u_c &= 0, \quad (11a) \\
\lambda_c &= \sum_{i \in I_H} B_{Ci,i} \mu_i. \quad (11b)
\end{align}

Thus, there is no cooling involved in Case I. Simplifying equation (9b) using complimentary slackness condition (8c), it is evident that $\lambda_i = 0 \forall i \in I$. The local control law can be written as:

\begin{align}
u_i &= \frac{B_i}{2} \mu_i = \frac{T - A_i T_i - \hat{d}_i}{B_i} \forall i \in I_H. \quad (12a) \\
u_i &= 0 \forall i \notin I_H. \quad (12b)
\end{align}

In order for $u_i \geq 0$ and $\mu_i \geq 0 \forall i \in I_H$, it is necessary that $T - A_i T_i - \hat{d}_i \geq 0$ (note that $B_i > 0$). Also, if $T - A_i T_i - \hat{d}_i = 0 \forall i \in I_H$, the lower temperature bound is weakly active $\forall i \in I_H$ and the control law corresponds to the Case IV control law.

**Remark 2.** Intuitively, the optimality region of the Case I control law can be thought of as a non-cooling region because $u_i \geq 0$ and $u_c = 0$. The zones $i \in I_H$ must request heat because they would violate the lower temperature bound on $T_i(k+1)$ if $u_c = u_i = 0$.

**B. Case II: Active Upper Bound Temperature Constraints and No Active Lower Bound Temperature Constraint**

In Case II we have:

\begin{align}
\check{\mu}_i &= 0 \forall i \in I \quad (13a) \\
\bar{\mu}_i &= 0 \forall i \in I \setminus I_C \quad (13b) \\
A_i T_i + B_{Ci}u_c + B_i u_i + \hat{d}_i &= T \forall i \in I_C. \quad (13c)
\end{align}

Following a similar approach to Case I, we obtain:

\begin{align}
u_c &= -\frac{(A_i T_i + \hat{d}_i - T)}{B_{Ci,i}} \forall i \in I_C \quad (14a) \\
u_i &= 0 \forall i \in I. \quad (14b)
\end{align}

Note that $u_c$ must be the same $\forall i \in I_C$ and $(A_i T_i + \hat{d}_i - T) \geq 0$ if $i \in I_C$. If $u_c = 0$, the constraints
Remark 3. Intuitively, this region is a non-heating region because \( u_c \leq 0 \) and \( u_i = 0 \\) \( \forall i \in \mathcal{I} \).

C. Case III: Active Upper Bound Temperature Constraints and Active Lower Bound Temperature Constraints

In Case III we have:

\[
\tilde{\mu}_i = 0 \quad \forall i \in \mathcal{I} \setminus \mathcal{I}_C \tag{15a}
\]

\[
A_i T_i + B_{C,i} u_c + B_i u_i + \tilde{d}_i = \bar{T} \quad \forall i \in \mathcal{I}_C \tag{15b}
\]

\[
\mu_i = 0 \quad \forall i \in \mathcal{I} \setminus \mathcal{I}_H \tag{15c}
\]

\[
A_i T_i + B_{C,i} u_c + B_i u_i + \tilde{d}_i = T \quad \forall i \in \mathcal{I}_H. \tag{15d}
\]

By following the same approach as Case I and Case II, we can compute the optimal control law for Case III.

\[
u_c = -\frac{(A_i T_i + \tilde{d}_i - \bar{T})}{B_{C,i}} \quad \forall i \in \mathcal{I}_C \tag{16a}
\]

\[
u_i = 0 \quad \forall i \in \mathcal{I} \setminus \mathcal{I}_H \tag{16b}
\]

\[
u_i = \frac{T - A_i T_i - \tilde{d}_i - B_i u_c}{B_i} \quad \forall i \in \mathcal{I}_H. \tag{16c}
\]

Remark 4. Intuitively, this region can be thought of as a mixed heating and cooling region because \( u_c \leq 0 \) and \( u_i \geq 0 \) \( \forall i \in \mathcal{I}_H \).

D. Summary of Distributed Feedback Control Algorithm

For all cases, it is convenient to define an initial guess for the dual variables \( \mu_i \) and \( \tilde{\mu}_i \) as follows:

\[
\mu_0 = (\mathbb{T} - A_i T_i - \tilde{d}_i)_+ \tag{17a}
\]

\[
\tilde{\mu}_0 = (-\mathbb{T} + A_i T_i + \tilde{d}_i)_+ \tag{17b}
\]

where \((x)_+ := \max(x, 0)\). Zone \( i \) requests heat if \( \mu_0 > 0 \), and likewise, \( \tilde{\mu}_0 > 0 \) indicates that zone \( i \) requests cooling. In fact, given parameter values \( T_i \) and \( \tilde{d}_i \) for all \( i \in \mathcal{I} \), the sets \( \mathcal{I}_C \) and \( \mathcal{I}_H \) can be determined as follows:

\[
\mathcal{I}_C = \left\{ i \in \mathcal{I} | \arg \min_i \frac{-\mu_0}{B_{C,i}} \right\} \tag{18}
\]

where \( \mathcal{I}_C = \emptyset \) if the minimizing value is zero, and

\[
\mathcal{I}_H = \left\{ i \in \mathcal{I} | \mu_0 - B_{C,i} u_c > 0 \right\}. \tag{19}
\]

The zones in \( \mathcal{I}_C \) correspond to the zones requesting the maximum cooling required for any zone to maintain the upper temperature bound constraint (8i). Similarly, the zones in \( \mathcal{I}_H \) request heat in order to maintain the lower temperature bound constraint (8j), assuming \( u_c \) has been calculated. Case I has \( \mathcal{I}_C = \emptyset \), Case II has \( \mathcal{I}_H = \emptyset \), and Case IV has \( \mathcal{I}_C = \mathcal{I}_H = \emptyset \).

Because the optimal law for \( u_c \) is determined from a sorting of \( \tilde{\mu}_0 \) and does not rely on \( u_i \), only one local-to-central and one central-to-local communication is required. Therefore, the structure of this problem allows us to design a dual decomposition algorithm which converges in one step. The one-step DMPC solution is provided in Algorithm 1. Throughout the algorithm, it is assumed that each zone only knows its own model parameters \( (A_i, B_{C,i}, B_i, \tilde{d}_i) \), and the central processor knows \( B_{C,i} \). The keyword “Local” indicates a computation at the VAV processor, and the keyword “Central” indicates a computation at the AHU processor.

**Algorithm 1 Explicit Solution of one-step DMPC**

**Computation of initial dual guess:**

1: Local: Compute \( \mathcal{I}_C \) from (18)
2: if \( \mathcal{I}_C == \emptyset \) then
3: \( \text{Central: } u_c := 0 \)
4: else
5: \( \text{Central: Compute } u_c \) from (16a)
6: end if
7: Send \( u_c \) to local VAV processors,
8: Local: Compute if \( i \in \mathcal{I}_H \) from (19)
9: if \( i \in \mathcal{I}_H \) then
10: \( \text{Local: Compute } u_i \) from (16c)
11: \( \text{else} \)
12: \( \text{Local: } u_i := 0. \)
13: \( \text{end if} \)

Remark 5. The Explicit DMPC solution presented in this Section IV is a fixed point solution of the dual-decomposition algorithm [9]. In fact, at each time step \( k \) the iterative dual-decomposition algorithm converges to one of the optimal controllers (Cases I-IV) presented in Section IV.

V. TRIM AND RESPOND

A well-regarded example of distributed control logic used in buildings is the “Trim and Respond” logic [10]. In this logic, each reheating input \( u_i(k) \) is controlled locally by a PI loop regulating zone temperature to remain within comfort bounds. Each zone generates a heating request when the reheating coil is approaching its maximum capacity. This maximum capacity is denoted as \( \bar{u}(k) \) and is changed over time by the “Trim and Respond” logic. Likewise, each zone generates a cooling request when the zone is close to its lower thermal bound. Since there is no cooling capacity at each zone, the zone must request that the central cooling input \( u_c(k) \) add more cool air to the system, and thus \( u_c(k) \) is also modulated over time by the “Trim and Respond” logic as a function of cooling requests.

Next, we provide more details about the “Trim and Respond” logic. A heating request \( r_H^i(k) \) is generated by zone \( i \) at time step \( k \) if its local reheating \( u_i(k) \) is near its maximum capacity \( \bar{u}(k) \). Similarly, zone \( i \) generates a cooling request \( r_c^i(k) \) if its local reheating \( u_i(k) \) is near zero and the temperature is close to its upper bound \( \bar{T} \). Each zone has an importance multiplier \( M_i \in \mathbb{Z}_{>0} \) which can be manually increased for a zone to have more influence on central control. The following formulas describe request
Remark 6. The logic presented in this section is a simplified version of “Trim and Respond” logic used in buildings today. In particular, we have only considered heating and cooling requests. In practice, the logic extends to pressure control as well.

VI. COMPARISON OF METHODS

The explicit MPC method and the “Trim and Respond” method lead to controllers with the same structure and similar data communication. A detailed explanation is provided next.

The initial dual variables and requests have the same meaning: initial dual variables are a real-valued measurement of temperature bound violation (17), whereas requests (20) are integer-valued indicators of zone heating or cooling demand. While dual variables are the one-step predicted heating or cooling demand, requests are manually tuned to demand an appropriate amount of central control adjustment. The “Trim and Respond” algorithm handles this coarse measurement of demand by ignoring some requests (using req.) and setting a maximum change in centralized control, e.g. max_c to prevent overreaction. If the model is accurate, the dual variables provide more valuable information than the boolean requests values.

The “Trim and Respond” importance factor \( M_i \) attempts to manually find the zone that has the highest cooling demand. Typically, one zone tends to request the most cooling (e.g. a computer server room or other zone with regularly high levels of thermal load), and so it is given the highest importance factor. The MPC algorithm automatically finds this zone by calculating \( I_C \).

Even if requests are only a function of the current state \( T(k) \), predictive capability is introduced in practice. Requests are generated when local controls are close to their limits instead of at saturation. Thus, requests are generated when temperature bounds are likely to be saturated in the near future.

The explicit MPC algorithm and the “Trim and Respond” logics communicate between processors in a similar fashion. “Trim and Respond” sends information from local processors to centralized processors. Communication from central to local processors is not present because each local PI controller will implicitly respond to changes in \( u_c \). Explicit MPC requires one communication each way between local and central processors (see Algorithm 1).

Both methods address energy efficiency. MPC directly addresses this by minimizing an effective cost of energy use. “Trim and Respond” addresses energy consumption by using a “trim” factor to decrease the control setpoints.

Finally, “Trim and Respond” does not require load estimation or a system dynamics model. However, there are many parameters to manually tune in the “Trim and Respond” algorithm.

VII. RESULTS: SIMULATIONS

Simulation results are presented in this section for the Model Predictive Control algorithm and the “Trim and Respond” algorithm. The simulation is run with \( n = 10 \) zones. The MPC has a sampling time of 5 minutes; the “Trim and Respond” logic updates the centralized controls \( u_c(k) \), \( \bar{u}(k) \) every 5 minutes with zone-level PI loops running
at 1 minute intervals. Model parameters from equation (1) are: \( C_i = 9.2 \text{e}3 \text{kJ}/\text{C} \), \( R_i = 50 \text{C}/\text{kW} \), \( T = 21 \text{C} \), and \( T = 26 \text{C} \).

The disturbance \( d_i \) is shown in Figure 2, estimated from data from the UC Berkeley Bancroft Library. Simulation results are provided in Figure 3. The MPC implementation uses a Luenberger observer to estimate the load, allowing a slight violation of temperature bounds. The “Trim and Respond” profile shows many small variations in temperature, while MPC provides a smoother result. The small variations are an artifact of the integer requests and the prefixed resolution by which central controls can change in the “Trim and Respond” logic. Similar comparisons exist between the control inputs of the two methods as well as between the MPC initial dual variables and the “Trim and Respond” requests. A calculation of total power used illustrates that the explicit MPC algorithm on hardware.

VIII. Conclusion

In this paper, we have demonstrated two ways to approach the design of distributed control in building HVAC systems. A one-step DMPC was formulated and explicitly solved by using KKT conditions. It was then compared to an existing control strategy, “Trim and Respond”. Simulation results of both algorithms are very similar. Future work includes adding control saturation to the explicit solution and implementing the explicit MPC algorithm on hardware.

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